Weak Turbulence in Ocean Waves

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LONG-TERM GOALS

Further develop weak turbulence theory, which is used to predict the evolution of spectral energy density in ocean surface waves and internal waves. Isolate, identify and quantify sources of possible discrepancies between numerical solutions of weak turbulence modeling and observations of ocean waves. Obtain theoretical predictions of forms of steady state spectral energy distributions for surface and internal ocean waves.

OBJECTIVES

(A) Apply weak turbulence methodology using rigorous mathematical techniques to identify possible sources of discrepancies between theory and observation of surface gravity waves.

(B) Calculate corrections to the surface wave kinetic equations arising from these discrepancies.

(C) Construct the form of the stationary spectral energy density of wind driven surface gravity waves in view of the modified kinetic equation.

(D) Use weak turbulence theory to predict the stationary spectral energy density of internal waves in the ocean. Compare the results with experimental observations.

(E) Study the coupling between ocean surface and internal waves, in particular the coupling of long gravity waves to internal waves.

APPROACH

Weak (or wave) turbulence is a universal theory used for the statistical description of an ensemble of weakly interacting waves. It has been used for the description of ocean waves since pioneering works of Hasselmann[1] and Zakharov[2-3].

The key feature of the weak turbulence description is the derivation of the statistical equation for the time evolution of spectral energy density of wave fields. Such an equation is called a kinetic equation. Derivation of the kinetic equation is in turn based on the Hamiltonian structure of the waves in question and multiple time scale expansions of time evolution equations of the statistical averages.

Some key assumptions used for the derivation of the kinetic equations are:

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14. ABSTRACT

Further develop weak turbulence theory, which is used to predict the evolution of spectral energy density in ocean surface waves and internal waves. Isolate, identify and quantify sources of possible discrepancies between numerical solutions of weak turbulence modeling and observations of ocean waves. Obtain theoretical predictions of forms of steady state spectral energy distributions for surface and internal ocean waves.

15. SUBJECT TERMS

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(A)Weak nonlinear interactions between waves.

(B) Random phases of the waves.

(C) Exactly resonant interactions between wave triads, quartets or quintets.

Violation of any of these assumptions or other implicit assumptions used to derive kinetic equations will lead to discrepancies between theory and experiments.

The project approach includes

(1) *Setting stage for use of weak turbulence theory*. For example, to rigorously apply weak turbulence formalism to internal waves, we had to develop from scratch Hamiltonian formalism for internal waves. In our Hamiltonian formulation the resulting field equations are *equivalent* to the dynamical equations, whereas previously used approaches relied on a small displacement approximation to derive similar field equations. Therefore our form of internal wave equations preserves all the necessary symmetries of the original dynamical equations, like energy conservation, incompressibility, etc.

(2) *Further development of weak turbulence theory*. For example, weak turbulence theory in its traditional formation totally ignores the spectral fluctuations, yet important statistical information is contained in these fluctuations. Understanding these fluctuations is of crucial importance to ONR, as these fluctuations may be responsible for "rare" events.

(3) *Application of weak turbulence theory for description of ensembles of nonlinear interacting waves, like ocean surface waves and internal waves*. In the process of doing so it may be necessary to modify the usual kinetic equation to take into account previously neglected processes.

(4) *Constant contact with observationalists to verify the resulting theorie*s.

Principal collaborators on this project are my graduate student Boris Pokorni, Prof. Esteban Tabak from Courant Institute and Dr. Kurt Polzin from WHOI. I also collaborate with Prof. Sergey Nazarenko from the University of Warwick (UK) on general wave turbulence methodology. I have started to collaborate with Prof. Peter Muller from the University of Hawaii.

WORK COMPLETED

(A) In the previous year we have systematically derived the kinetic equation appropriate for the description of wave-wave interaction of internal waves. After the kinetic equation for internal waves was derived, the next task is to derive its steady state solutions, as those would correspond to the spectral energy density observable in the ocean. In our previous works [5-6] we have analytically found one such exact steady state solution, henceforth denoted as LT, $n(\mathbf{k},m) \propto |\mathbf{k}|\hat{}(-7/2)m\hat{}(-1/2)$. This LT spectrum turns out to be not far away from the high frequency – high wave number limit of the celebrated Garrett-and-Munk spectrum of internal waves, $n(\mathbf{k},m) \propto |\mathbf{k}|\hat{}(-4)m\hat{}(0)$. Yet there is a small difference that motivated the work presented below. We have assumed that the kinetic equation derived in [5-6] has not one, but rather a family of steady state solutions of the form $n(\mathbf{k},m) \propto |\mathbf{k}|\hat{}(-x)m\hat{}(-y)$. We have then

substituted this expression into the kinetic equation, and have developed an efficient method to integrate the kinetic equation numerically. By doing so we have obtained the *curve* in (x, y) plain of steady state solutions of the kinetic equation of internal waves in the ocean. It turns out that there is a region in (x, y) plain where the resulting kinetic equation *converges,* so that such distributions may be in fact realized in the ocean. Therefore one might expect that depending upon specific parameters (to be identified) of the experiment, one member of the family of steady state solutions will be realized. This family of steady state solutions is presented below as a black curve on the figure.

(B) Remarkably, this theoretical curve passes through the high-frequency-high-wave number limit of the Garrett-and-Munk spectrum of internal waves. Therefore we have shown, for the first time, that the high-frequency-high-wave number limit of the Garrett-Munk spectrum of internal waves constitutes an exact steady state solution of the weak turbulence kinetic equation.

(C) Together with Kurt L. Polzin from WHOI we have reviewed the oceanographic literature and analyzed the high-frequency-high-wave number parts of the measured internal waves energy spectra. Seven such observational sets were analyzed: NATRE, AIWEX, FASINEX, MODE, PATCHEX, SWAPP and IWEX. It became increasingly apparent, that these deep ocean observations exhibit a larger degree of variability than one might anticipate for a universal spectrum. Moreover, the deviations from the Garrett-and-Munk power laws form a pattern: they seem to roughly fall upon a curve with negative slope in (x, y) plane.

(D) Remarkably, the pattern of observed variations of internal wave spectrum is *consistent* with that predicted in (A).

(E) We have also analyzed the steps used for deriving the kinetic equation. As explained above, one of the key assumptions used for the derivation of the kinetic equation is the random phase approximation. While analyzing the Random Phase approximation it became increasingly apparent that the wave-action alone, does not give enough statistical information about the form of the interacting wave field. However, traditional wave turbulence theory made a considerable effort to ignore spectral fluctuations. We have generalized the weak turbulence formalism to include the spectral fluctuations, and therefore derived the hierarchy of closed evolution equation for *K'th* spectral moment. Traditional kinetic equation is just a first equation in this hierarchy. This is very important result, because it predicts the level of spectral energy fluctuations to be at the Gaussian level in the steady state. Moreover, our theory gives theoretical prediction for the rate of spectral fluctuations grow or decay in surface waves in the ocean in the event of variable forcing, for example due to local changes in wind strength or direction. .

RESULTS

Previous results:

- The validity of Zakharov's form of the weak turbulence Hamiltonian was reestablished
- The form of the surface-wave Hamiltoniian in physical space was established, and it was found to coincide with generalization of Choi's Hamiltonian [4]. The resulting Hamiltonian is simpler and more compact than the one in [3].
- The equivalence of these two Hamiltonian structures was established.

• The canonical Hamiltonian structure for long internal waves in hydrostatic balance in a rotating environment was derived [6].
• The weak turbulence formalism was generalized for a simple model to include kinetic equations arising from near-resonant interaction of triads of waves [7].
• The kinetic equation for the spectral energy density evolution of internal waves was found, and its solution in the high frequency limit was derived. This solution is not far from the Garrett and Munk spectra of internal waves in the ocean [6].

This year results:

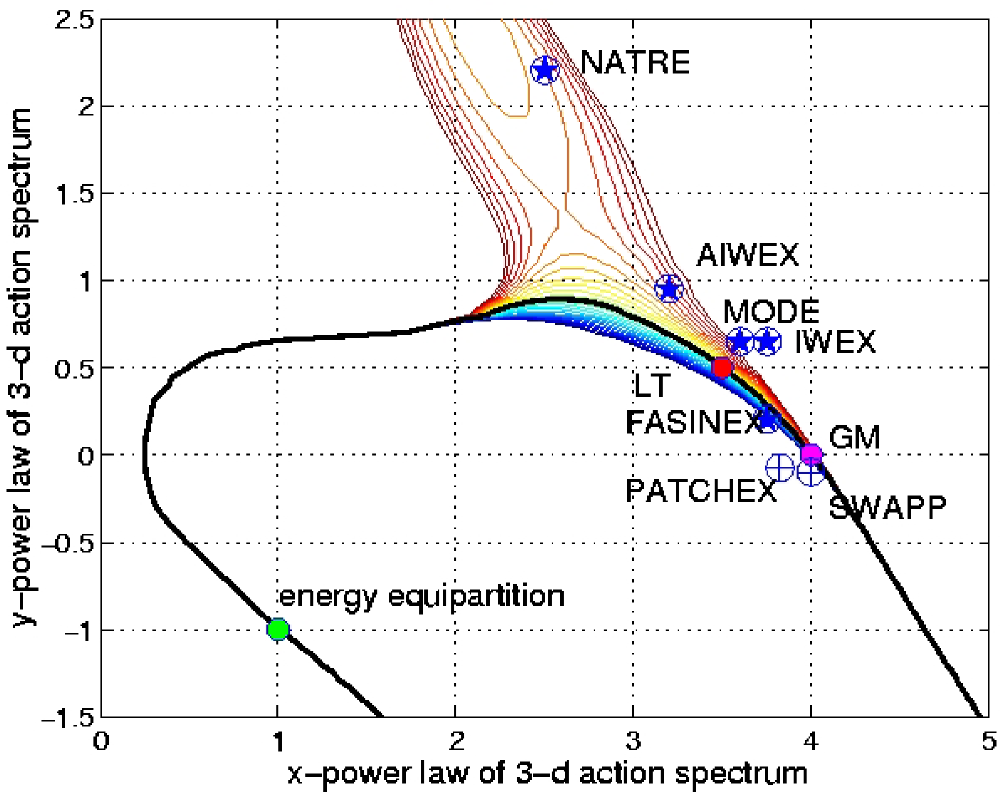

Figure 1: Ocean observations and analytical zeroes of the kinetic equation in the (x, y) ***plane with the high-frequency-high-wave action spectrum given by the power-law. Solid red dot represents the closed form-zero LT solution of the kinetic equation, that we have obtained analytically earlier, solid magenta dot represents the high-frequency-high-wave number limit of the Garrett-Munk spectrum of internal waves in the ocean, and the solid green dot represents the thermodynamic equilibrium of the kinetic equation (mostly irrelevant in the context of the ocean). Blue circles represent different observational sets. The solid black curve marks the numerically computed zeros of the kinetic equation. Contour lines of the kinetic equation are also shown, with read curves corresponding to positive and blue curve to negative values of the RHS of kinetic equation. Note that observations (blue circles) form a pattern, which is in agreement with the theory (black curve). Note also that the theoretical curve passes exactly through the high-frequency limit of the Garrett-and-Munk spectrum of internal waves, thus demonstrating for the first time that this limit of the GM spectrum constitutes exact steady state solution of the internal wave kinetic equation.***

- It was shown numerically that the kinetic equation describing Nonrotating Ocean has not one, but rather a family of steady state solutions [9] (see figure above).
- It was shown that the high-frequency-high-wave number limit of the celebrated Garrett-and-Munk spectrum of internal waves in the ocean constitutes *an exact* steady state solution of the kinetic equation of internal waves.
- The high-frequency-high wavenumber limit of the observed internal wave spectra were re-analyzed and it was established that the deviation from the Garrett-and-Munk spectrum of internal waves form a pattern
- This pattern can be explained by the weak turbulence theory as described above.
- Weak turbulence theory was generalized to include spectral fluctuations of the interacting wavefields. Prediction for the rate of growth of spectral fluctuations was obtained for surface tension waves.

IMPACT/APPLICATIONS

Continuing results from this project will significantly enhance our understanding of nonlinear wave interactions in shallow- and deep-water environments and consequently will lead to improved forecasting and prediction for Naval and civilian applications.

TRANSITIONS

RELATED PROJECTS

REFERENCES

[1]K.Hasselmann, "On the nonlinear energy transfer in gravity wave spectrum", J. Fluid Mech, **12**, 481, (1962), **15,** 273, (1962).

[2]V.E. Zakharov, "Stability of periodic waves of finite amplitude on a surface of deep fluid", J. Appl. Mech. Tech.Phys., **2,** 190 , (1967); "The instability of waves in nonlinear dispersive media", Sov. Phys. JETP, **24,** 740, (1967).

[3]V.E. Zakharov, V.S.Lvov and G.Falkovich, "Kolmogorov Spectra of Wave Turbulence", Springer-Verlag, (1992).

[4] W. Choi, " Nonlinear evolution equations for two-dimensional surface waves in a fluid of a finite depth", J. Fluid Mech., **295,** 381, (1995).

[5]Yuri V. Lvov and Esteban G. Tabak, "Hamiltonian formalism and the Garrett-Munk spectrum of internal waves in the ocean", Phys. Rev. Lett, **87,** 168501, (2001).

[6] Yuri V. Lvov and Esteban G. Tabak, "A Hamiltonian Formulation for Long Internal Waves", submitted to Physica D, (2003).

[7] J. Biello, P. Kramer and Y. Lvov, "Applications of weak turbulence theory to models with almost resonant interactions”, to appear in the journal of Discrete and Continious Dynamical Systems. 2003

[8] Yuri V Lvov and Sergei Nazarenko, "Statistics of the spectral fluctuations in wave turbulence", submitted to Physical Review Lettrers

[9] Yuri V Lvov, Kurt L Polzin and Esteban G. Tabak, "Energy spectra of the ocean's internal wave field: theory and observations", submitted to Physical Review Letters, 2003.

PUBLICATIONS

- Published: [5] and [7].
- Submitted: [6], [8], [9].